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The gluon propagator in the Coulomb gauge

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Abstract. We give the results for all the one-loop propagators, including finite parts, in the Coulomb gauge. In the finite parts we find new non-rational functions in addition to the single logarithms of the Feynman gauge. Of course, the two gauges must agree for any gauge invariant function.

1 Introduction

The non-covariant axial and Coulomb gauges have a more direct physical interpretation than the covariant gauges, because their propagators are closely related to the polarization states of real spin-1 particles. The relevant diagrams in the Coulomb gauge are not plagued by ghosts. Also the time-time component of the gluon propagator provides a long-range confining force [1,2]. The Hamiltonian for non-Abelian gauge theory in the Coulomb gauge has been known for some time in its continuum version [3]. The Coulomb gauge in the Hamiltonian formalism is manifestly unitary. The main point in its favor is that problems concerned with the definition of the axial gauge integrals like

$$\int \mathrm{d}^4 k \frac{1}{(n\cdot k)^2} \tag{1}$$

do not appear in the definition of integrals like

$$\int \mathrm{d}^4 k \frac{1}{K^2}\cdots \tag{2}$$

in the Coulomb gauge. However, there are disadvantages. The naive Coulomb gauge Feynman rules in non-Abelian gauge theory give rise to ambiguous integrals, in addition to the usual ultraviolet divergences [4]. At one loop order and above there are integrals like

$$\int \frac{\mathrm{d}^3 P}{(2\pi)^3}\int \frac{\mathrm{d}p_0}{(2\pi)}\frac{p_0}{p_0^2-P^2+\mathrm{i}\eta}\times\frac{1}{(P-K)^2}. \tag{3}$$

There is no regularization procedure for the energy divergence in p_0 within the standard dimensional regularization scheme. This integral and similar more complicated divergences in higher order diagrams have been the subject of study [5,6], where systematic cancellations have been found. However, no general proof exists that controls all divergences [7]. Formally such integrals are assigned the value zero. The Coulomb gauge has been extensively studied in the phase space formalism by Zwanziger [8] in the Euclidean space. The ultraviolet divergent parts of the proper two-point functions have been calculated and found to observe the Ward identities. In addition, a more powerful Ward identity holds in the Coulomb gauge than is available in covariant gauges. In this paper we give the results for the complete propagator to order g^2 including finite parts in Minkowski space.

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2 The Coulomb gauge in the phase-space formalism

We use the phase-space formalism in order to avoid the ambiguous integrals like (3). Let the generating functional of the Green's functions be

$$Z(j,J) = \int \mathrm{d}[f]\mathrm{d}[A][J^\mu A_\mu + j^{\mu\nu} f_{\mu\nu}]\exp\left[-\mathrm{i}\int \mathrm{d}^4 x L\right], \tag{4}$$

where J, j are sources, L the Lagrangian density and f and A are the fields [9];

$$L = -\frac{1}{4} f^a_{\mu\nu} f^{a\mu\nu} + \frac{1}{2} f^{a\mu\nu} F^a_{\mu\nu} \tag{5}$$

and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu. \tag{6}$$

Greek indices run from 0 to 3 and Latin indices denote spatial dimensions ($i = 1,2,3$); a, b, c, are color indices. Instead of setting the source j to zero as in the Lagrangian formalism, we keep some components of j. We write out L as

$$L = -\frac{1}{4}(f^a_{ij})^2 + \frac{1}{2}(f^a_{0i})^2 + \frac{1}{2} f^a_{ij} F^a_{ij} - f^a_{0i} F^a_{0i} \tag{7}$$

and set $j_{ij} = 0$ in order to perform the Feynman integral over f_{ij}. We denote $f_{0i} = E_i$. The Lagrangian becomes

$$L = -\frac{1}{4}(F^a_{ij})^2 + \frac{1}{2}(E^a_i)^2 - E^a_i F^a_{0i}, \tag{8}$$

where

$$E^a_i F^a_{0i} = E^a_i[\partial_0 A^a_i - \partial_i A^a_0 - g f^{abc} A^b_0 A^c_i]. \tag{9}$$

The field E_i is the momentum conjugate to A_i. We have L expressed in terms of momenta and linear terms in the derivatives. It is first order in the time derivatives. Adding the gauge fixing term

$$\frac{1}{2\alpha}(\partial_i A_i)^2, \tag{10}$$

we can deduce the propagators from the quadratic part of the Lagrangian. The propagator matrix is the inverse of the matrix of these quadratic parts and is given by the following 7×7 matrix, whose rows and columns are labeled by A_1, A_2, A_3 ; A_0; E_1, E_2, E_3:

	A_j	A_0	E_n
A_i	$-T_{ij}/k^2 + \alpha L_{ij}/K^2$	$\alpha k_0 K_i/(K^2)^2$	$-\mathrm{i}k_0 T_{in}/k^2$
A_0	$\alpha k_0 K_j/(K^2)^2$	$1/K^2 + \alpha k_0^2/(K^2)^2$	$\mathrm{i}K_n/K^2$
E_m	$-\mathrm{i}k_0 T_{mj}/k^2$	$\mathrm{i}K_m/K^2$	$-T_{mn}K^2/k^2$

where

$$T_{ij} \equiv \delta_{ij} - L_{ij}, \quad L_{ij} \equiv K_i K_j/K^2, \\ k^2 = k_0^2 - K^2. \tag{11}$$

The Coulomb gauge propagators are obtained by setting $\alpha = 0$.

3 The proper two-point functions

The method of evaluation of Coulomb gauge integrals is explained in Appendix A. Here we list the results. The constants used throughout this paper are

$$\epsilon = 4 - d, \tag{12}$$

where d is the dimension of space-time and the coupling parameter is

$$c\delta_{ab} = \frac{\mathrm{i}g^2}{16\pi^2} C_G \delta_{ab}. \tag{13}$$

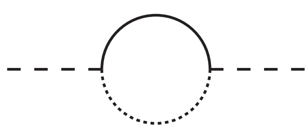


Fig. 1. The transverse gluon self-energy graph. The dashed line is the transverse gluon A_i, the dotted line represents the instantaneous Coulomb field A_0 and the continuous line is the E_i field conjugate to the transverse field A_i

The transverse gluon two-point function

There are two non-vanishing graphs contributing to the transverse gluon propagator. The graph shown in Fig. 1 gives

$$\Gamma_1^{A_i A_j} = c\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{K^2}{\mu^2}\right)^{-\frac{\epsilon}{2}} \times 2^{-\epsilon}\left(1 + \frac{77}{30}\epsilon\right) \\ \times \left\{\frac{12}{15}K^2\delta_{ij} - \frac{16}{15}K_i K_j + \frac{4}{15}\epsilon K_i K_j\right\}. \tag{14}$$

The graph in Fig. 2 contributes

$$\Gamma_2^{A_i A_j} = c(M K_i K_j + N K^2 \delta_{ij}), \tag{15}$$

where

$$\begin{aligned} M = {} & \frac{31}{15}\Gamma\left(\frac{\epsilon}{2}\right) - \frac{4}{3}\ln\left(\frac{-k^2 - \mathrm{i}\eta}{\mu^2}\right) + \frac{3}{10}\ln\frac{K^2}{\mu^2} \\ & + \frac{1}{4}\left[-\frac{K^4}{k_0^2} + 18\frac{k_0^2 k^2}{K^2} + 9\frac{k_0^2 k^4}{K^4} + k_0^2\right] \times D \\ & + \frac{1}{4}\left[-\frac{K^3}{k_0^3} + 18\frac{k_0 k^2}{K^3} + 9\frac{k_0 k^4}{K^5} + \frac{k_0}{K}\right] \ln\frac{k_0 + K - \mathrm{i}\eta}{k_0 - K + \mathrm{i}\eta} \\ & \quad \times \ln\frac{K^2}{(-k^2 - \mathrm{i}\eta)} \\ & - \frac{1}{2K^2}\left[3k_0^2 - 5K^2 + 9\frac{k_0^4}{K^2} - \frac{K^4}{k_0^2}\right]\ln\frac{K^2}{(-k^2 - \mathrm{i}\eta)} \\ & - \frac{\ln 2}{K^2}\left[6\frac{k_0^4}{K^2} + 9k^2 - \frac{1}{5}K^2 + 3\frac{k^4}{K^2} - \frac{K^4}{k_0^2}\right] \\ & + 22\frac{k_0^2}{K^2} - 16 - \frac{1}{9} - \frac{8}{15} + 4 \times \frac{77}{225}, \end{aligned} \tag{16}$$

$$\begin{aligned} N = {} & -\frac{1}{3K^2}\left(k_0^2 + \frac{27}{5}K^2\right)\Gamma\left(\frac{\epsilon}{2}\right) \\ & + \frac{1}{3K^2}(k_0^2 + 8K^2)\ln\left(\frac{-k^2 - \mathrm{i}\eta}{\mu^2}\right) - \frac{13}{15}\ln\frac{K^2}{\mu^2} \\ & + \frac{1}{4}\left[\frac{K^4}{k_0^2} - \frac{k_0^2 k^2}{K^2}\left(14 + \frac{K^2}{k^2} + 3\frac{k^2}{K^2}\right)\right] \times D \\ & + \frac{1}{4}\left[\frac{K^3}{k_0^3} - \frac{k_0 k^2}{K^3}\left(14 + \frac{K^2}{k^2} + 3\frac{k^2}{K^2}\right)\right] \ln\frac{k_0 + K - \mathrm{i}\eta}{k_0 - K + \mathrm{i}\eta} \\ & \quad \times \ln\frac{K^2}{(-k^2 - \mathrm{i}\eta)} \\ & + \frac{1}{2K^2}\left(9k^2 + 6k_0^2 + K^2 + 3\frac{k^4}{K^2} - \frac{K^4}{k_0^2}\right) \\ & \quad \times \ln\frac{K^2}{(-k^2 - \mathrm{i}\eta)} \end{aligned}$$

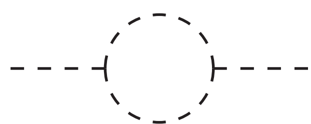


Fig. 2. The transverse gluon self-energy graph. The dashed line is the transverse gluon field A_i

$$+\frac{\ln 2}{K^2}\left(9k^2+6k_0^2+3\frac{k^4}{K^2}-\frac{K^4}{k_0^2}-\frac{11}{15}K^2\right)$$
$$-16\frac{k_0^2}{K^2}+10+\frac{2}{9}. \tag{17}$$

The non-rational structure D which appears in the results for the proper two-point functions is in the integral form

$$D=\int_0^1 \mathrm{d}x\frac{x^{-\frac{1}{2}}}{k_0^2-x(K^2-\mathrm{i}\eta)}\ln(1-x). \tag{18}$$

In the region $k_0 > K$

We have in the region $k_0 > K$

$$D=\frac{1}{k_0K}\left\{\mathrm{Li}_2\left(\frac{k_0-K+\mathrm{i}\eta}{k_0+K-\mathrm{i}\eta}\right)-\mathrm{Li}_2\left(\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta}\right)\right.$$
$$+\ln\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta}\times\ln\frac{k^2+\mathrm{i}\eta}{K^2}$$
$$\left.-\mathrm{i}\pi\ln\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta}\right\}. \tag{19}$$

In the region $K > k_0$

We have in the region $K > k_0$

$$D=\frac{1}{k_0K}\left\{\mathrm{Li}_2\left(\frac{K-k_0-\mathrm{i}\eta}{K+k_0-\mathrm{i}\eta}\right)-\mathrm{Li}_2\left(\frac{K+k_0-\mathrm{i}\eta}{K-k_0-\mathrm{i}\eta}\right)\right.$$
$$+\ln\frac{K+k_0-\mathrm{i}\eta}{K-k_0-\mathrm{i}\eta}\times\ln\left(\frac{-k^2-\mathrm{i}\eta}{k_0^2}\right)$$
$$\left.+\mathrm{i}\pi\ln\frac{K+k_0-\mathrm{i}\eta}{K-k_0-\mathrm{i}\eta}\right\}$$
$$-\frac{2}{k_0K}\left[\mathrm{Li}_2\left(-\frac{k_0}{K-\mathrm{i}\eta}\right)-\mathrm{Li}_2\left(\frac{k_0}{K-\mathrm{i}\eta}\right)\right]$$
$$+\frac{\mathrm{i}\pi}{k_0K}\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)}, \tag{20}$$

where

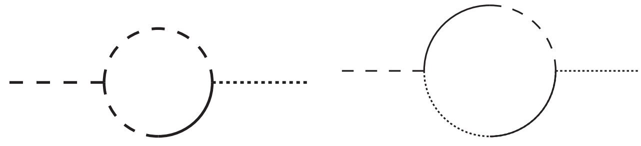
$$\mathrm{Li}_2(x)=-\int_0^x\frac{\ln(1-z)}{z}\mathrm{d}z \tag{21}$$

is the Spence function and k_0 and K are the lengths of the respective vectors. The two expressions for D in (19) and (20) are connected as analytic continuations of each other with the relation (B1).

The A_iA_0 transition

The whole contribution to the A_iA_0 transition to order g^2 comes from the graph in Fig. 3a. We have

$$\Gamma^{A_iA_0}=ck_0K_i\times Z, \tag{22}$$



a b

Fig. 3. **a** The A_iA_0 two-point function. The dotted line is the instantaneous Coulomb field A_0, the dashed line represents the transverse field A_i and the solid line is the conjugate field E_i. **b** The A_iA_0 two-point function. The graph is suppressed as an energy divergence

$$Z=-\frac{1}{3}\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{-k^2-\mathrm{i}\eta}{\mu^2}\right)^{-\frac{\epsilon}{2}}$$
$$+\frac{k^2}{2K^2}(2k_0^2+k^2)\times D$$
$$+\frac{k^2}{2k_0K^3}(2k_0^2+k^2)\ln\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta}\times\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)}$$
$$-3\frac{k^2}{K^2}\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)}-6\frac{k^2}{K^2}\ln 2+6\frac{k_0^2}{K^2}-\frac{53}{9}. \tag{23}$$

The graph shown in Fig. 3b contains integrals like the one in (3). Formally such integrals are assigned the value zero.

The time-time component of the two-point function

Two graphs contribute to the A_0A_0 function. The graph shown in Fig. 4a gives

$$\Gamma_a^{A_0A_0}=c\left\{\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{-k^2-\mathrm{i}\eta}{\mu^2}\right)^{-\frac{\epsilon}{2}}\right.$$
$$\times\left(\frac{1}{2}k_0^2+\frac{5}{6}K^2+\frac{\epsilon}{12}k^2+\frac{\epsilon}{6}k_0^2+\frac{17}{18}\epsilon K^2\right)$$
$$-2^{-\epsilon}\left(\frac{5}{3}+\frac{28}{9}\epsilon\right)\Gamma\left(\frac{\epsilon}{2}\right)K^2\left(\frac{K^2}{\mu^2}\right)^{-\frac{\epsilon}{2}}$$
$$+\frac{1}{2}k^4\times D+\frac{k^4}{2k_0K}\ln\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta}\times\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)}$$
$$\left.-k_0^2\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)}-2(\ln 2-1)k_0^2\right\}. \tag{24}$$

a b

Fig. 4. **a** The time-time component of the gluon self-energy to order g^2. The dotted lines represent the instantaneous Coulomb field A_0. The continuous line is the E_i field conjugate to the transverse field A_i. The propagators inside the loop are the E_iA_j transitions specific to the Coulomb gauge. **b** Self-energy graph to order g^2. The dotted line is the A_0 field, the dashed line is the transverse propagator and the solid line is the E_iE_j propagator

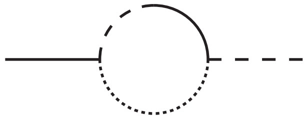


Fig. 5. The transition between the transverse gluon field and its conjugate field E_i

The graph in Fig. 4b contributes

$$\begin{aligned}\Gamma_b^{A_0A_0} = c\Big\{ &\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{-k^2-\mathrm{i}\eta}{\mu^2}\right)^{-\frac{\epsilon}{2}} \\ &\times\left(\frac{1}{3}K^2-\frac{1}{2}k^2-\frac{\epsilon}{4}k^2+\frac{11}{18}\epsilon K^2\right) \\ &-\frac{1}{3}\Gamma\left(\frac{\epsilon}{2}\right)K^2\left(\frac{K^2}{\mu^2}\right)^{-\frac{\epsilon}{2}}-K^2\left(\frac{10}{9}-\frac{2\ln 2}{3}\right) \\ &+k^2k_0^2\times D+\frac{k_0k^2}{K}\ln\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta}\times\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)} \\ &-(2k^2+K^2)\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)} \\ &-2(2k^2+K^2)(\ln 2-1)\Big\}\,. \end{aligned} \tag{25}$$

We can verify that the complete proper two-point functions satisfy the 't Hooft identity [10]

$$k_0^2\Gamma^{A_0A_0}-2k_0K_i\Gamma^{A_iA_0}+K_iK_j\Gamma^{A_iA_j}=0 \tag{26}$$

and the stronger Zwanziger identity [8]

$$k_0\Gamma^{A_0A_0}=K_i\Gamma^{A_iA_0}. \tag{27}$$

The remaining graphs contain the conjugate field E_i as the external leg.

E_iA_j graph

The graph in Fig. 5 vanishes as the energy diverges.

E_iA_0 graph

The graph in Fig. 6 gives

$$\begin{aligned}\Gamma^{E_iA_0} = c\left(\frac{K^2}{\mu^2}\right)^{-\frac{\epsilon}{2}}&\left\{\frac{4}{3}-2^{-\epsilon}\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{2}{3}+\frac{13\epsilon}{9}\right)\right\} \\ &\times(2\mathrm{i}K_i). \end{aligned} \tag{28}$$

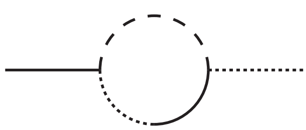


Fig. 6. The transition between the Coulomb field A_0 and the conjugate field E_i

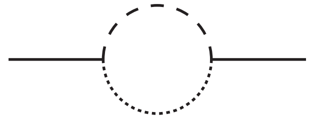


Fig. 7. The conjugate field self-energy

E_iE_j graph

The graph in Fig. 7 contributes

$$\begin{aligned}\Gamma^{E_iE_j} &= -2c\left(\frac{K^2}{\mu^2}\right)^{-\frac{\epsilon}{2}} \\ &\times\left\{2^{-\epsilon}\left(\frac{2}{3}+\frac{13\epsilon}{9}\right)\Gamma\left(\frac{\epsilon}{2}\right)\delta_{ij}-\frac{4}{3}\frac{K_iK_j}{K^2}\right\}. \end{aligned} \tag{29}$$

4 The gluon propagator to order g^2

We form the 7×7 matrix of free and order g^2 proper two-point functions. The inverse of this matrix gives the gluon propagator to order g^2.

The A_0A_0 propagator

The time-time component of the gluon propagator to order g^2 is [11]

$$\begin{aligned}D^{A_0A_0} &= \frac{1}{K^4}[\Gamma^{A_0A_0}+\mathrm{i}K_n\Gamma^{A_0E_n}] \\ &+\frac{\mathrm{i}K_m}{K^4}[\Gamma^{E_mA_0}+\mathrm{i}K_n\Gamma^{E_mE_n}], \end{aligned} \tag{30}$$

or explicitly

$$\begin{aligned}&D^{A_0A_0} \\ &= c(K^2)^{-2}\times\Big\{\frac{11}{3}\Gamma\left(\frac{\epsilon}{2}\right)K^2-\frac{5}{3}K^2\ln\frac{(-k^2-\mathrm{i}\eta)}{\mu^2} \\ &-2K^2\ln\frac{K^2}{\mu^2}+\frac{1}{2}k^2(k^2+2k_0^2)\times D \\ &+\frac{k^2}{2k_0K}(k^2+2k_0^2)\ln\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta}\times\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)} \\ &-(3k_0^2-K^2)\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)} \\ &-(6k_0^2+2K^2)\ln 2+6k_0^2+\frac{31}{9}K^2\Big\}\,. \end{aligned} \tag{31}$$

The ultraviolet divergent part of (31) gives the gauge invariant Coulomb field renormalization factor [12].

The A_iA_j propagator

The transverse gluon propagator to order g^2 is

$$D^{A_iA_j}=\frac{1}{k^4}T_{am}\Gamma^{A_mA_n}T_{nj}-\frac{k_0^2}{k^4}T_{am}\Gamma^{E_mE_n}T_{nj} \tag{32}$$

or explicitly

$$\begin{aligned}
&D^{A_iA_j} \\
&= \frac{c}{k^2+\mathrm{i}\eta}\left(\delta_{ij}-\frac{K_iK_j}{K^2}\right) \\
&\qquad \times\left\{\Gamma\left(\frac{\epsilon}{2}\right)-\frac{4}{3}\ln\frac{K^2}{\mu^2}+\frac{1}{3}\ln\left(\frac{-k^2-\mathrm{i}\eta}{\mu^2}\right)\right. \\
&-\frac{K^2}{4}\left[\frac{K^2+k_0^2}{k_0^2}+\frac{k_0^2}{K^2}\left(14+3\frac{k^2}{K^2}\right)\right]\times D \\
&-\frac{K}{4k_0}\left[\frac{K^2+k_0^2}{k_0^2}+\frac{k_0^2}{K^2}\left(14+3\frac{k^2}{K^2}\right)\right]\ln\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta} \\
&\qquad \times\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)} \\
&+\frac{1}{2}\left(15+3\frac{k^2}{K^2}+\frac{K^2}{k_0^2}\right)\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)} \\
&+\left.\left(3\frac{k^2}{K^2}+\frac{K^2}{k_0^2}+\frac{37}{3}\right)\ln 2-\frac{92}{9}\right\}. \qquad (33)
\end{aligned}$$

5 The Slavnov–Taylor identity

Although ghosts are absent from the S-matrix elements they are necessary to formulate the Slavnov–Taylor identities [13,14]. Diagramatically they are shown for the self-energy in Fig. 8. Algebraically they are

$$k_0\Gamma^{A_0A_j}-K_i\Gamma^{A_iA_j}=(K^2\delta_{ij}-K_iK_j)\Gamma^{CA_i}. \qquad (34)$$

The diagrams involving ghost–source vertices on the right-hand side are shown in Fig. 9a,b. The diagram in Fig. 9a vanishes as the energy divergence in p_0. The diagram in Fig. 9b contributes

$$\begin{aligned}
\Gamma^{CA_i} &= -2c\left(\frac{K^2}{\mu^2}\right)^{-\frac{\epsilon}{2}} \\
&\quad\times K_i\left\{-\frac{4}{3}+2^{-\epsilon}\Gamma\left(\frac{\epsilon}{2}\right)\left(\frac{2}{3}+\frac{13\epsilon}{9}\right)\right\}, \qquad (35)
\end{aligned}$$

so the identity is satisfied trivially as implied by (26) and (27).

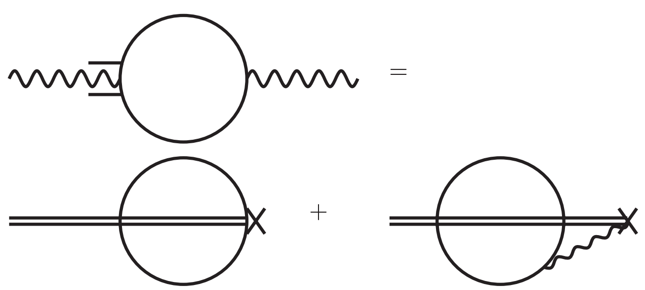


Fig. 8. The Slavnov–Taylor identity for self-energy graphs. The wavy lines stand for Yang–Mills particles and double lines for ghosts. The symbol on the left wavy line stands for the replacement of a polarization vector $e_\mu(k)$ by k_μ and k^2 need not be zero. The cross denotes the action of the tensor $(k_\mu k_\nu - k^2\delta_{\mu\nu})$. The circle represents the set of all relevant Feynman graphs

6 Discussion

We have checked the consistency of the Coulomb gauge to order g^2 including finite parts. The time-time component of the gluon propagator in the Coulomb gauge is believed to provide a long-range confining force. There are two interesting limits of (31). In the Zwanziger picture [8] g^2D_{00} gives the instantaneous part $V_Z(R)$, which is called the color-Coulomb potential. (Here D_{00} is the time-time component of the gluon propagator.) The instantaneous color-Coulomb potential $V_Z(R)$ at large R may serve as an order parameter. We have

$$K_{\mathrm{Coul}}\equiv\lim_{R\to\infty}\frac{V_Z(R)}{R}. \qquad (36)$$

A non-zero value of K_{Coul} would be the signal for color confinement. The potential is separated out in momentum space by

$$V_Z(K)=\lim_{k_0\to\infty}g^2D^{A_0A_0}(k_0,K), \qquad (37)$$

where we have written $V_Z(K)$ for the Fourier transform of $V_Z(R)$. The limit $k_0\to\infty$ of (31) is

$$\begin{aligned}
&\lim_{k_0\to\infty}D^{A_0A_0}(k_0,K) \\
&\quad=\frac{c}{K^2}\left\{\frac{11}{3}\Gamma\left(\frac{\epsilon}{2}\right)-\frac{11}{3}\ln\frac{K^2}{\mu^2}-\mathrm{i}\pi\right. \\
&\qquad\left.-\frac{28}{3}\ln 2+\frac{103}{9}-2\ln\frac{K}{k_0}\right\}, \qquad (38)
\end{aligned}$$

and it is not independent of k_0. There appears to be a difference in the dominant term in (38) and (37) of Cucchieri and Zwanziger [15]. This difference arises because of the statement near the end of Appendix B in [15] that I_2 is finite, and "as a result" I_2 vanishes in the limit $k_0\to\infty$. However, the finiteness of I_2 does not imply anything about the behavior as $k_0\to\infty$. In fact, on calculating I_2, we find that the dominant term as $k_0\to\infty$ is $-4/3\ln(k_0^2/K^2)$. With this value, there is no contradiction with (38) in this paper.

Although the limit as $k_0\to\infty$ is not finite, Cucchieri and Zwanziger [15] have argued that an unambiguous instantaneous part may be defined by using renormalization group arguments.

The limit $k_0\to 0$ is naturally related to the definition of the quark–antiquark potential. It follows from considering a rectangular Wilson loop with sides of length T in

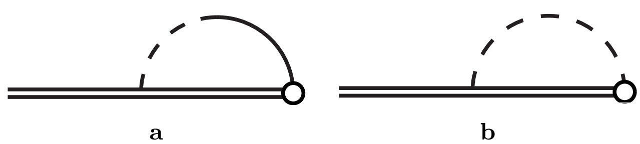


Fig. 9. a Diagram with an open ghost line. The source v_n of the E_m field has the vertex $gf^{abc}E_n^bC^cv_n$. The ghost propagator is $\frac{1}{K^2}$ and it is represented with the double line. **b** Diagram with an open ghost line. The source u_i^a of the transverse gluon field has the vertex $gf^{abc}\delta_{ij}$

the time direction (where $T \to \infty$) and L in the space direction. In the Coulomb gauge the main contribution comes from the D_{00} component of the propagator (where $k_0 \to 0$) attached to the two time-like sides. The $k_0 \to 0$ limit of (31) is

$$\lim_{k_0\to 0} D^{A_0A_0}(k_0,K) = \frac{c}{K^2}\left\{\frac{11}{3}\Gamma\left(\frac{\epsilon}{2}\right) - \frac{11}{3}\ln\frac{K^2}{\mu^2} + \frac{31}{9}\right\}, \tag{39}$$

leading to the quark–antiquark potential

$$V(R) = -2\pi^2 g_r^2(\mu) \times \frac{1}{R}\left\{1 + \frac{g^2 C_G}{16\pi^2}\left[\frac{31}{9} + \frac{11}{3}\gamma + \frac{11}{3}\ln(\mu R)^2\right]\right\}, \tag{40}$$

where γ is Euler's constant, and $g_r(\mu)$ is the running coupling constant. If we assume the relation

$$R \times \mu = 1, \tag{41}$$

$g_r(\mu)$ becomes R dependent. We suppose that the exact $g_r\left(\frac{1}{R}\right)$ tends to zero as $R \to 0$ and $g_r\left(\frac{1}{R}\right) \to \infty$ for $R \to \infty$.

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Appendix A

We use the following two basic integrals for the evaluation of the Coulomb gauge integrals:

$$A = \int d^{4-\epsilon}p \frac{1}{p^2 + i\eta}\cdot\frac{1}{(k-p)^2 + i\eta} = \frac{1}{2}i\pi\int_0^1 dy \int d^{3-\epsilon}P \times \{P^2 - 2P\cdot Ky - yk^2 + y^2k_0^2 - i\eta\}^{-\frac{3}{2}}, \tag{A1}$$

$$B = \int d^{4-\epsilon}p \frac{p_0}{(p^2 + i\eta)[(k-p)^2 + i\eta]} = \frac{1}{2}i\pi k_0\int_0^1 y dy \int d^{3-\epsilon}P \times \{P^2 - 2P\cdot Ky - yk^2 + y^2k_0^2 - i\eta\}^{-\frac{3}{2}}. \tag{A2}$$

As an example we evaluate the integral

$$X_{ij} = \int d^{4-\epsilon}p \frac{p_0}{p^2 + i\eta}\cdot\frac{1}{(k-p)^2 + i\eta}\cdot\frac{P_iP_j}{P^2}. \tag{A3}$$

Applying (A2)

$$X_{ij} = \frac{1}{2}i\pi k_0\int_0^1 y dy \int d^{3-\epsilon}P \frac{P_iP_j}{P^2}\cdot \times\frac{1}{[P^2 - 2P\cdot Ky - yk^2 + y^2k_0^2 - i\eta]^{\frac{3}{2}}}. \tag{A4}$$

Combining the denominators with the Feynman parameter x, we have

$$X_{ij} = ik_0\pi^{\frac{1}{2}}\Gamma\left(\frac{5}{2}\right)\int_0^1 dx x^{\frac{1}{2}}\int_0^1 y dy\int d^{3-\epsilon}P \times\frac{P_iP_j}{[P^2 - 2P\cdot Kxy - xyk^2 + y^2xk_0^2 - i\eta x]^{\frac{5}{2}}}. \tag{A5}$$

Now it is easy to perform the $d^{3-\epsilon}P$ integration and the integration over the parameter y giving X_{ij} in the integral form:

$$\begin{aligned} C^{-1}X_{ij} &= \frac{1}{6}\Gamma\left(\frac{\epsilon}{2}\right)(K^2)^{-\frac{\epsilon}{2}}\delta_{ij} - \frac{1}{3}\frac{K_iK_j}{K^2} + \frac{1}{3}\delta_{ij}\left(\frac{13}{6} - \ln 2\right) \\ &+ \left(\frac{1}{4}\delta_{ij} - \frac{K_iK_j}{K^2}\right)k^2\left\{\int_0^1 dx\frac{x^{\frac{1}{2}}}{k_0^2 - x(K^2 - i\eta)} + k^2\int_0^1 dx\frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^2}\ln\frac{K^2(1-x)}{(-k^2 - i\eta)}\right\} \\ &+ \frac{K_iK_j}{K^2}k_0^2\left\{\frac{1}{2}\int_0^1 dx\frac{x^{\frac{1}{2}}}{k_0^2 - x(K^2 - i\eta)} + k^2\int_0^1 dx\frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^2} + k^4\int_0^1 dx\frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^3}\ln\frac{(1-x)K^2}{(-k^2 - i\eta)}\right\}, \end{aligned}$$

where

$$C = ik_0\pi^{\frac{4-\epsilon}{2}}. \tag{A6}$$

The integrals in (A6) are

$$\int_0^1 dx\frac{x^{\frac{1}{2}}}{k_0^2 - x(K^2 - i\eta)} = -\frac{2}{K^2} + \frac{k_0}{K^3}\ln\frac{k_0 + K - i\eta}{k_0 - K + i\eta}, \tag{A7}$$

$$\int_0^1 dx\frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^2} = \frac{1}{k^2K^2} - \frac{1}{2k_0K^3}\ln\frac{k_0 + K - i\eta}{k_0 - K + i\eta}, \tag{A8}$$

$$\int_0^1 dx\frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^3} = \frac{1}{2K^2k^4} - \frac{1}{4k_0^2K^2k^2} - \frac{1}{8k_0^3K^3}\ln\frac{k_0 + K - i\eta}{k_0 - K + i\eta}, \tag{A9}$$

$$T = \int_0^1 dx\frac{x^{\frac{1}{2}}}{k_0^2 - x(K^2 - i\eta)} + k^2\int_0^1 dx\frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2 - i\eta)]^2}\ln\frac{K^2(1-x)}{(-k^2 - i\eta)}$$

$$= \frac{2}{K^2}(\ln 2 - 1) + \left[\frac{1}{K^2} - \frac{k^2}{2k_0K^3}\ln\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta}\right]\ln\frac{K^2}{(-k^2-\mathrm{i}\eta)} - \frac{k^2}{2K^2}D, \tag{A10}$$

$$E = \int_0^1 \mathrm{d}x \frac{x^{\frac{1}{2}}}{[k_0^2 - x(K^2-\mathrm{i}\eta)]^3}\ln(1-x) = \frac{k_0^2+K^2}{2k_0^2K^2k^4}\ln 2 - \frac{1}{2K^2k^4} - \frac{1}{2Kk_0k^4}\ln\frac{k_0+K-\mathrm{i}\eta}{k_0-K+\mathrm{i}\eta} - \frac{1}{8K^2k_0^2}D, \tag{A11}$$

where D was defined in (18) and the explicit result given in (19) and (20).

Appendix B

The expressions for D in (19) for $k_0 > K$ and in (20) for $K > k_0$ ought to be connected by analytic continuation. It is easy to see that this happens using the following relation between the Spence functions:

$$\mathrm{Li}_2\left(\frac{x-1+\mathrm{i}\eta}{x+1-\mathrm{i}\eta}\right) - \mathrm{Li}_2\left(\frac{x+1-\mathrm{i}\eta}{x-1+\mathrm{i}\eta}\right) + \mathrm{Li}_2\left(\frac{1+x-\mathrm{i}\eta}{1-x-\mathrm{i}\eta}\right) - \mathrm{Li}_2\left(\frac{1-x-\mathrm{i}\eta}{1+x-\mathrm{i}\eta}\right) + 2\mathrm{Li}_2(-x-\mathrm{i}\eta) - 2\mathrm{Li}_2(x+\mathrm{i}\eta) + \ln\frac{x+1-\mathrm{i}\eta}{x-1+\mathrm{i}\eta}\times\ln(x^2) - \mathrm{i}\pi\ln\frac{x+1-\mathrm{i}\eta}{x-1+\mathrm{i}\eta} + \mathrm{i}\pi\ln(x^2) + \pi^2 = 0, \tag{B1}$$

where

$$\mathrm{Li}_2(x) = -\int_0^x \frac{\ln(1-z)}{z}\mathrm{d}z. \tag{B2}$$

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